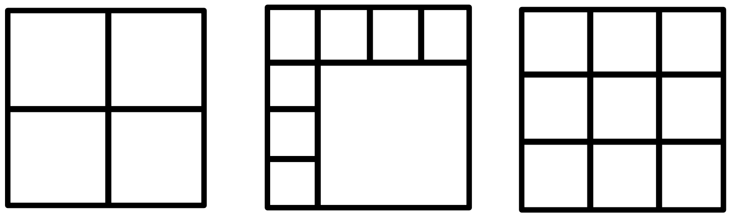
Ten Fantastic “Big Problems”

If you’re curious, we have scaffolded versions available for questions 1, 4, 7, and 8 – email us at malcolm.eckel@gmail.com and/or lynncp@beammath.org and we can send them to you.

1) Call a number “squareable” if you can cut a square into that many smaller, non-overlapping squares *not all of which must be the same size*. Which numbers are squareable and which aren’t? For example, 4, 8, and 9 are all squareable:



2) Write all of the numbers from 1 to 50 down in order. Can you write addition or subtraction signs in between all of the numbers so that the final sum is zero?

3) I’m thinking of a number. It’s divisible by every number from 1 to 31 except two of those numbers, and the two numbers it isn’t divisible by are *consecutive*. What are the two numbers it isn’t divisible by, and what’s the smallest number I could be thinking of?

4) Create a single closed polygon by tracing lines on a piece of graph paper. It can be as irregular and complex as you like, or something as simple as a square or rectangle, but there should be no holes and you must follow the lines on the graph paper. What perimeters are possible? What perimeters are impossible?

5) In Frobenia, only $5 and $7 bills are produced. What amounts of money can someone in Frobenia have? Extension: what if a shopkeeper can make change (ie, I pay two $5s and get $7 in return to purchase a $3 item) – what can an item cost? Bigger extension: pick your own two numbers that aren’t 5 and 7 and ask the previous two questions again. What patterns do you see?

6) Advanced kids often love playing around with different bases. My phrasing: At some point in history, someone first wrote down the symbols we use for the digits 0-9. They chose ten symbols because we have ten fingers. How would math be different if they only had eight fingers, and stopped at 7, deciding that 10 meant one group of eight instead of one group of ten? What would the place values be? How would you add, subtract, multiply, divide? Do algebra? Represent fractions?

7) In how many ways can you tile a 2 x n rectangle with 1 x 2 tiles? (For n = 1, the answer is 1; for n = 2, the answer is 2; for n = 3, the answer is 3; for n = 4, the answer is 5. These can be found fairly easily by drawing them. What about larger n?) For a more advanced version of this problem, do 3 x n and 1 x 3 instead. This is particularly awesome because there’s an obvious pattern that breaks; the answers go 1, 1, 2, 3, 4, ***6***. Every kid in the room will *know* it’s going to be 5 and be shocked!

8) I can write 35 as the sum of two consecutive numbers: 17 + 18. I can write 12 as the sum of three consecutive numbers: 3 + 4 + 5. Are there any numbers that *can’t* be written as the sum of some amount of consecutive numbers? Which numbers can be and which ones can’t? (This is a more difficult question, but has a stunningly beautiful answer.)

9) Imagine quadrant I of a coordinate plane – bounded at the left and below by the axes, but infinitely large above and to the right. Place a dot in the lower-left-corner square. You can, at any time, erase any dot and replace it with a dot in the square above it and a dot in the square to the right of it, but only if both of those squares are empty. Is it possible to clear the 3x3 square in the lower left corner? (This is a *substantially* more difficult question.)

10) Cut a square into triangles such that all the triangles are acute. Extension: cut a square into the *smallest possible number* of triangles that are all acute. (This problem is pure evil.)