BEAM
Bridge to Enter Advanced Mathematics

Supporting Strong Students in a Low-Income Classroom

Twitter: @BEAMmathHQ, Instagram: @BEAMmathNYC

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Getting to Know the Room

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“I like math because it’s easy.”

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Our work is urgent.

- National Assessment of Educational Progress (8th grade):

  "Advanced rating": only 2% of low-income students achieve it compared with 13% of non-low-income students.

- 12th grade: the percentage of low-income students scoring at the advanced level is so low it isn't even given. It "rounds to zero".

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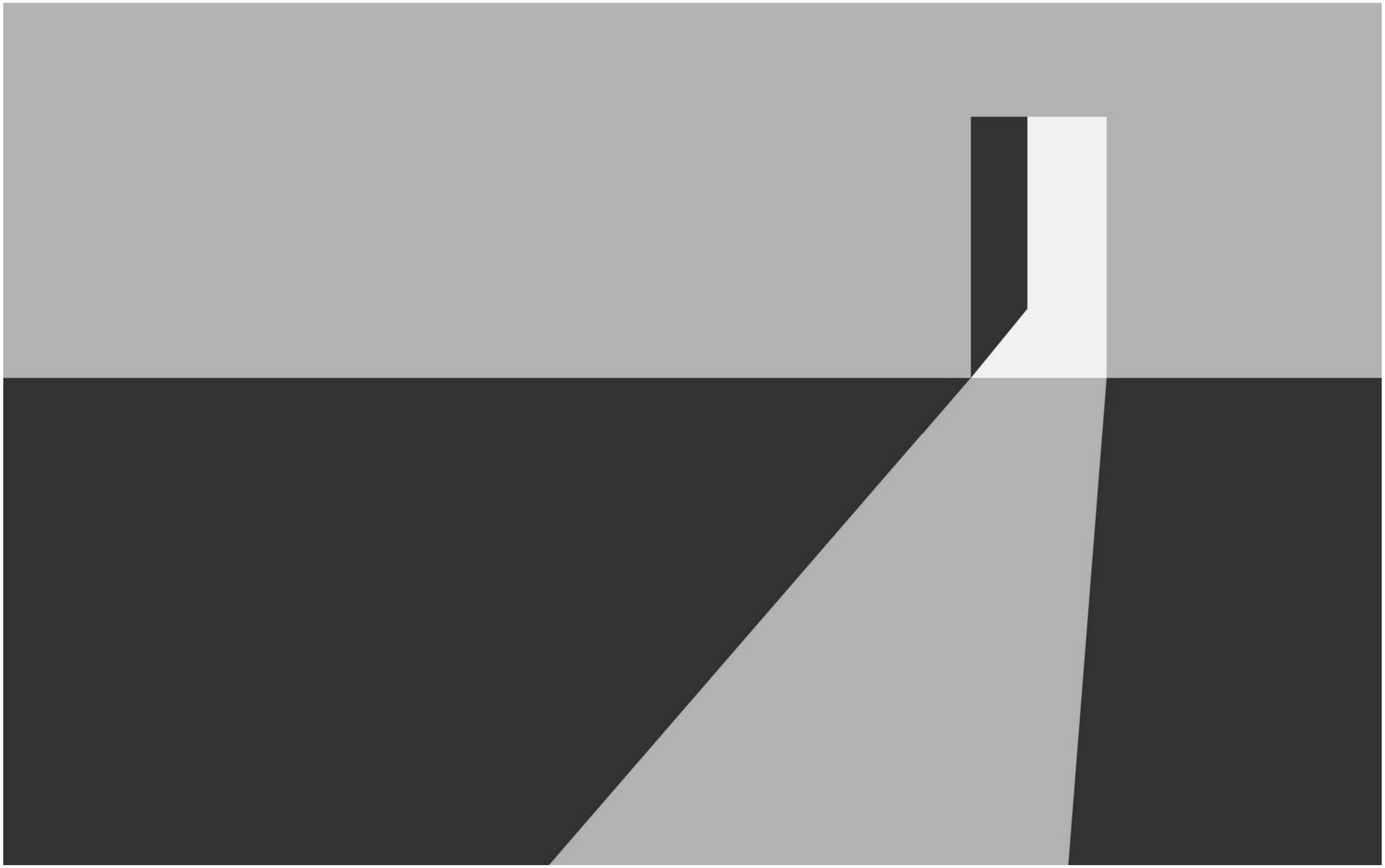
Students deserve options.



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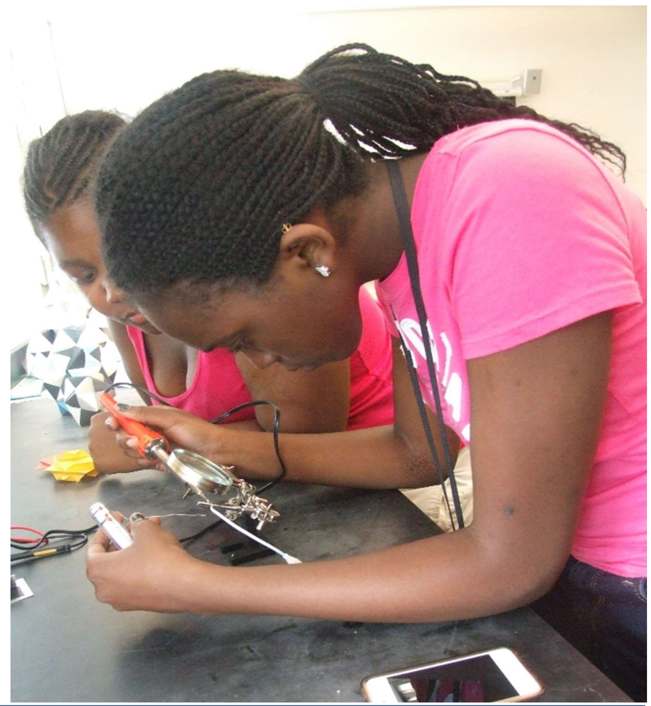
Acceleration is not the same as deep understanding.

- Aisha (12th grade)
- 4 on the AP Calc AB exam as an 11th grader
- Now in AP Calc BC
- Is she ready for college engineering classes?



Exam	# of Asian Students with a 5	# of Black Students with a 5
AP Calc AB	810	113
AP Calc BC	1,008	47

(Data May 2017, NY State Only)

(you might ask)

How do you have time?

Use the time you have.

- If you have 10 minutes…
- If you have 1-2 class periods…

- If you have time outside of class…
- If you have time to go down “the rabbit hole”…

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When you have 10 minutes...
Print Challenge Problems

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Foundations

Explore old math (in a new way).

Problem 5

Lianna makes two four-digit numbers using each of the digits 1, 2, 3, 4, 5, 6, 7, and 8 exactly once. If Lianna makes the numbers so that adding them gives the smallest possible total, what is that total?

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$$\begin{array}{r} 1357 \\ +\,2468 \\ \hline 3825 \end{array}$$

3825

+10

Lianna makes two four-digit numbers using each of the digits 1, 2, 3, 4, 5, 6, 7, and 8 exactly once. If Lianna makes the numbers so that adding them gives the smallest possible total, what is that total?

$$\begin{array}{r} 1357 \\ +\,2468 \\ \hline 3825 \end{array}$$

$$\begin{array}{r} 1234 \\ +\,5678 \\ \hline 5 \end{array}$$

+10

8 23

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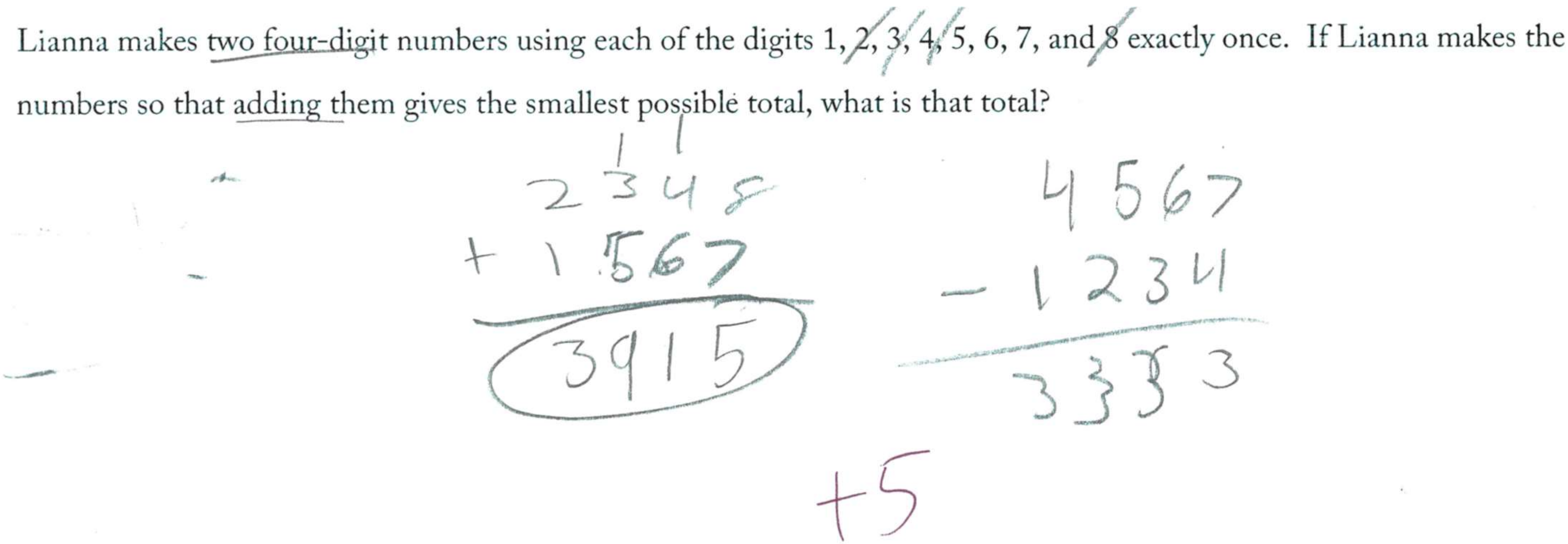
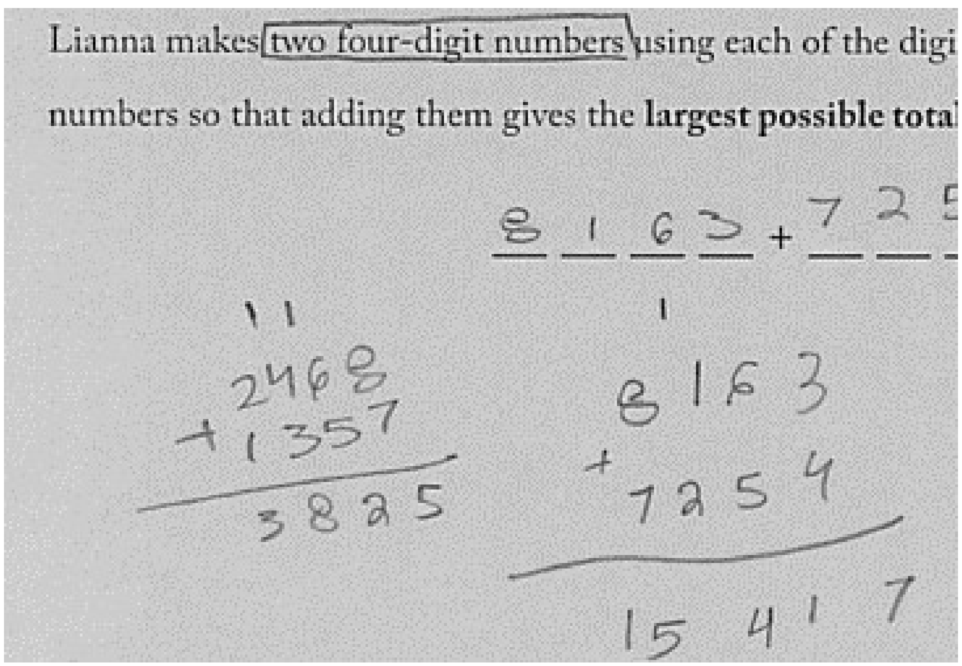
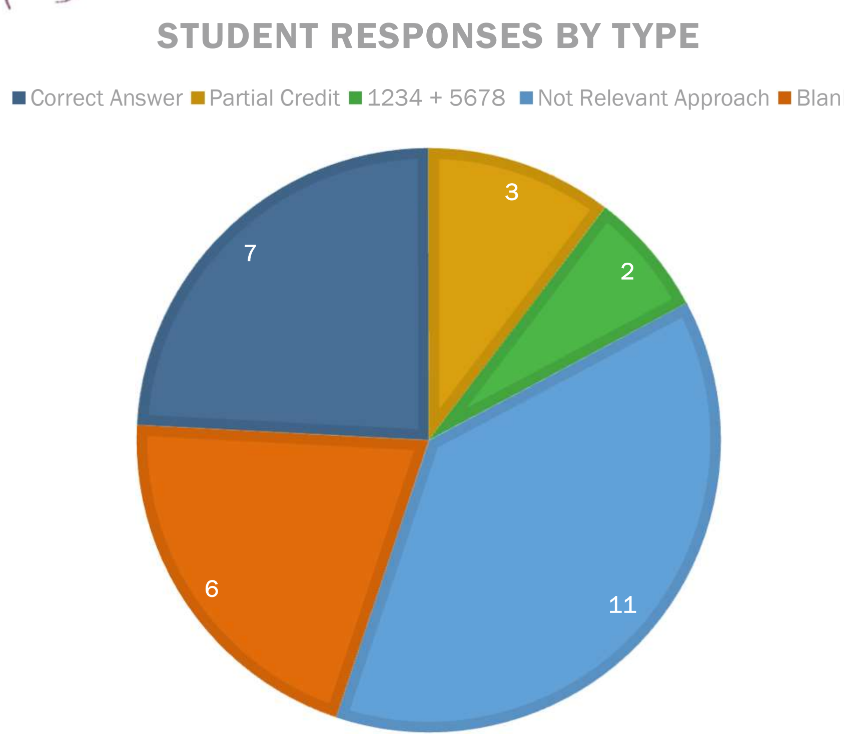

Lianna makes two four-digit numbers using each of the digits 1, 2, 3, 4, 5, 6, 7, and 8 exactly once. If Lianna makes the numbers so that adding them gives the smallest possible total, what is that total?
2348
+ 1567
3915
4567
- 1234
3333
+5


Lianna makes two four-digit numbers using each of the digi
numbers so that adding them gives the largest possible total
8163 + 725
2468
+ 1357
3825
8163
+ 7254
15417


STUDENT RESPONSES BY TYPE
Correct Answer
Partial Credit
1234 + 5678
Not Relevant Approach
Blank
3
2
7
6
11

When you have 1 or 2 periods…
Solve a big problem.

BEAM class:
Solving Big Problems

2 hours, 1 problem

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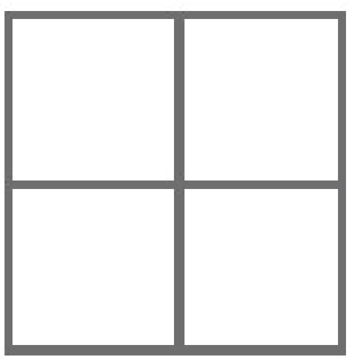
Enrichment

Explore math topics we generally don't have time for in K-12 classroom.

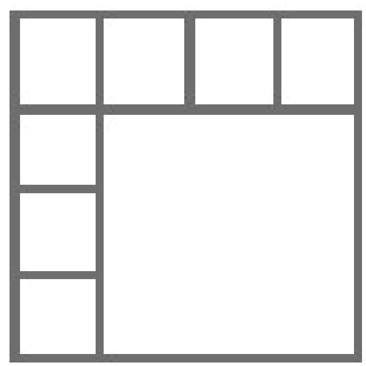
Problem 7 (spend some time playing with this!)

This question is a bit different and lets you explore more.

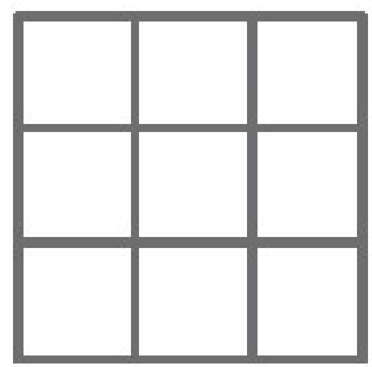
It is possible to divide a square into 4 squares, 8 squares, or 9 squares as pictured below:



4 squares



8 squares



9 squares

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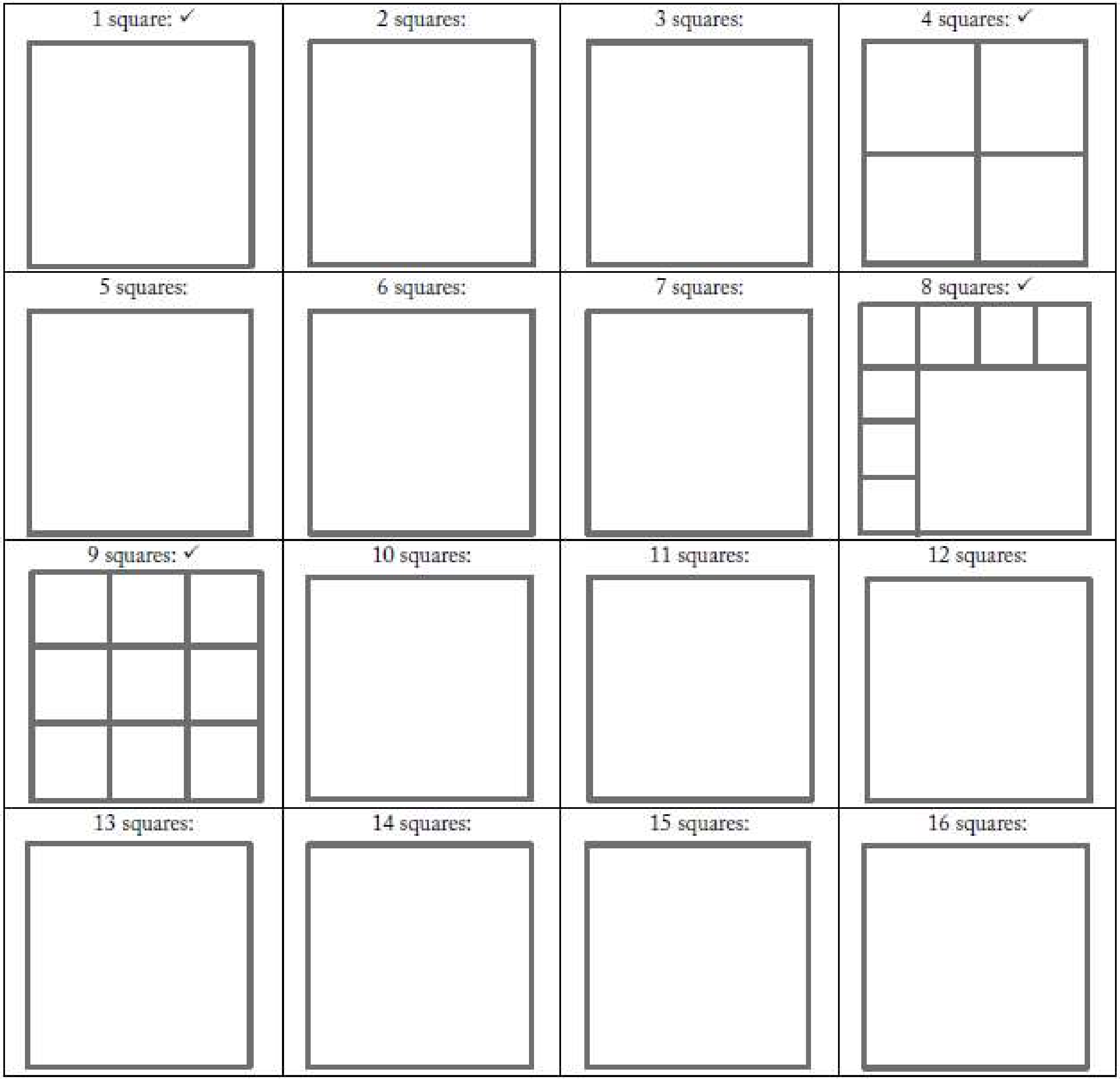

What do you notice?

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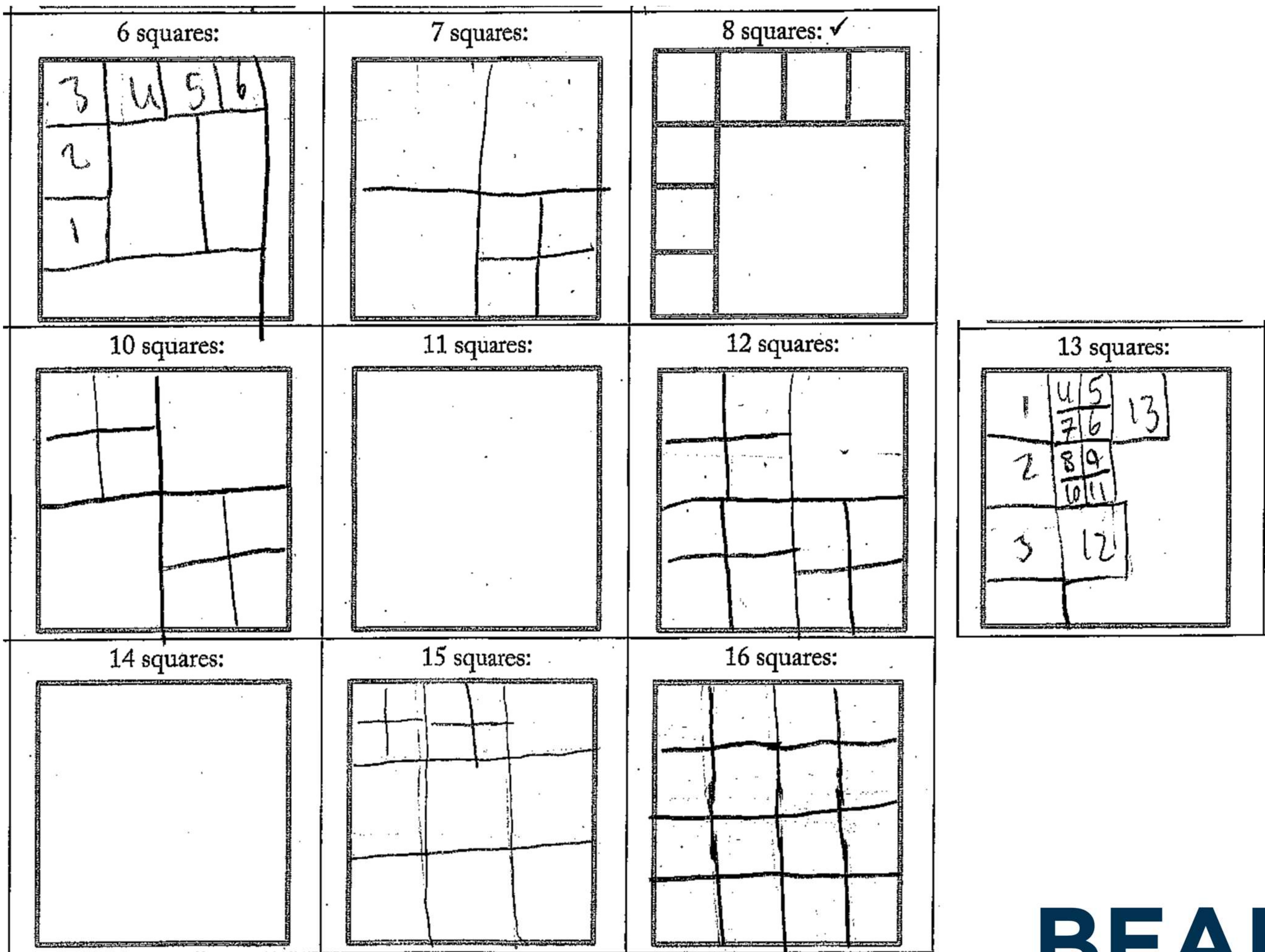

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Crystal's Theorem: We can make any even number n bigger than 2.

Proof: If I take the number $(2N)$ that I want to get and divide it by 2 (I get N), I know how big my square has to be: $N \times N$. I fill it out but then erase everything except the left side of squares and the top row of squares. There is 1 big square left in the middle. Then you count all the squares on the side and on top, making sure you don't recount any squares, and add that to the big square. That gives you $N + (N-1) + 1 = 2N$ squares total. In diagram B below, you can see how to do this for $N = 7$, to get $2N = 14$ squares. □

Proof written by Crystal.

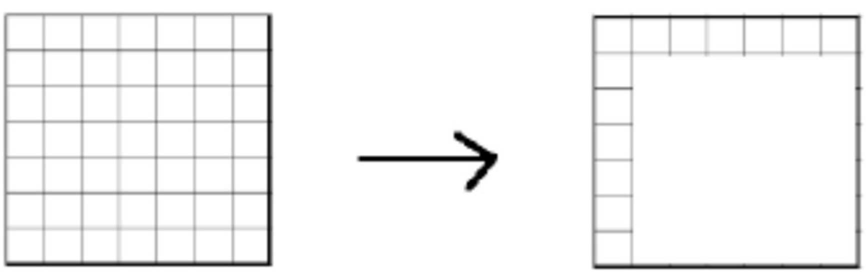
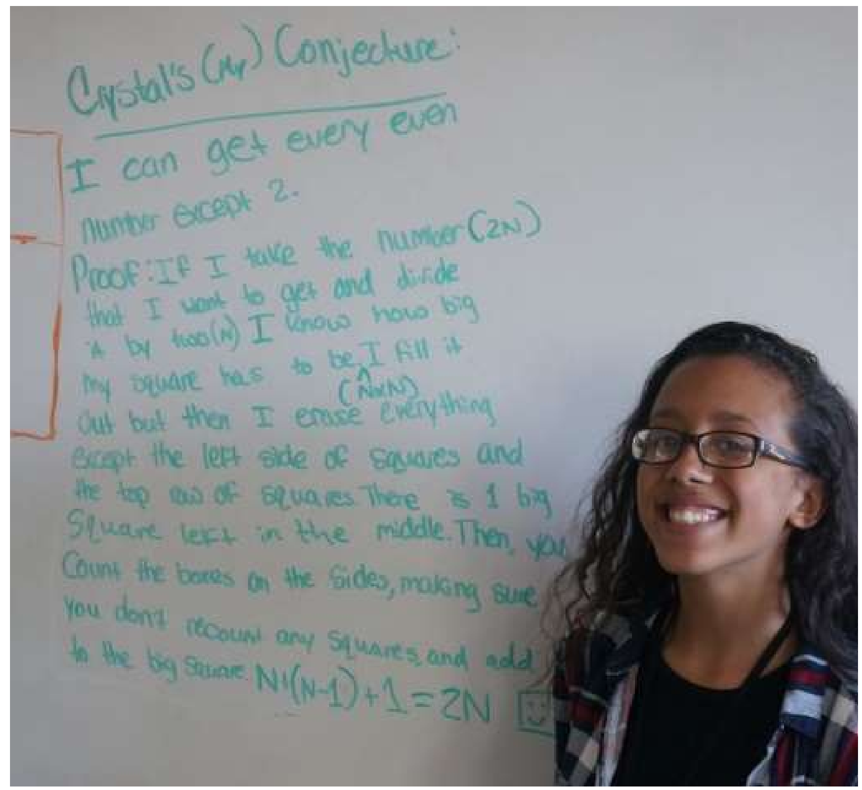


Diagram B: Crystal's Theorem



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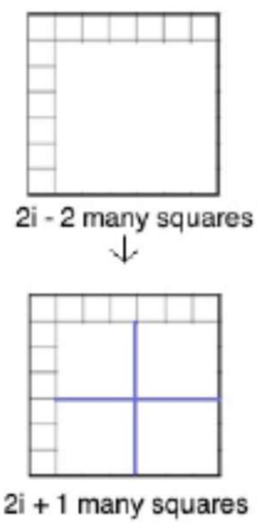
Gabby's Theorem: You can make any number n of squares if $n-3$ is possible.

Proof: Suppose you can divide a square into $n-3$ many smaller squares. Take any square in the $n-3$ square, and turn it into 4 squares (by cutting it in half horizontally and vertically). We took away one square but added 4, and $4-1=3$, so we now have $n-3+3=n$ squares. □

Jacob's Theorem: You can make any number of squares over 5.

Proof: We know we can make any even number over 5, from Michelle's Theorem.

If we want to make an odd number $n=2i+1$ squares, then we know that $2i+1-3=2i-2$ is an even number of squares. So by Michelle's theorem, we can divide a square into $2i-2$ many squares (as long as 2i-2 is at least 4). Then, we can take the big square in the picture of Michelle's Theorem and divide it into 4 by cutting it in half with a vertical and a horizontal line. You are getting rid of the big square, but adding 4 more, so you are adding 3 altogether. This means you will have $2i+1$ many squares in total (see the diagram below). This gives you all the ways to make odd n many squares, when n is at least 7. □



Conjecture (both classes): It is impossible to divide a square into 2, 3, or 5 smaller squares.

(you might state)

I don’t have time for anything that’s not in the standards.

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

COMMON CORE STATE STANDARDS for MATHEMATICS

(you might ask)

My principal gives me no flexibility; how can I make this work?

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When you have time beyond class...
Start a math circle or math team.

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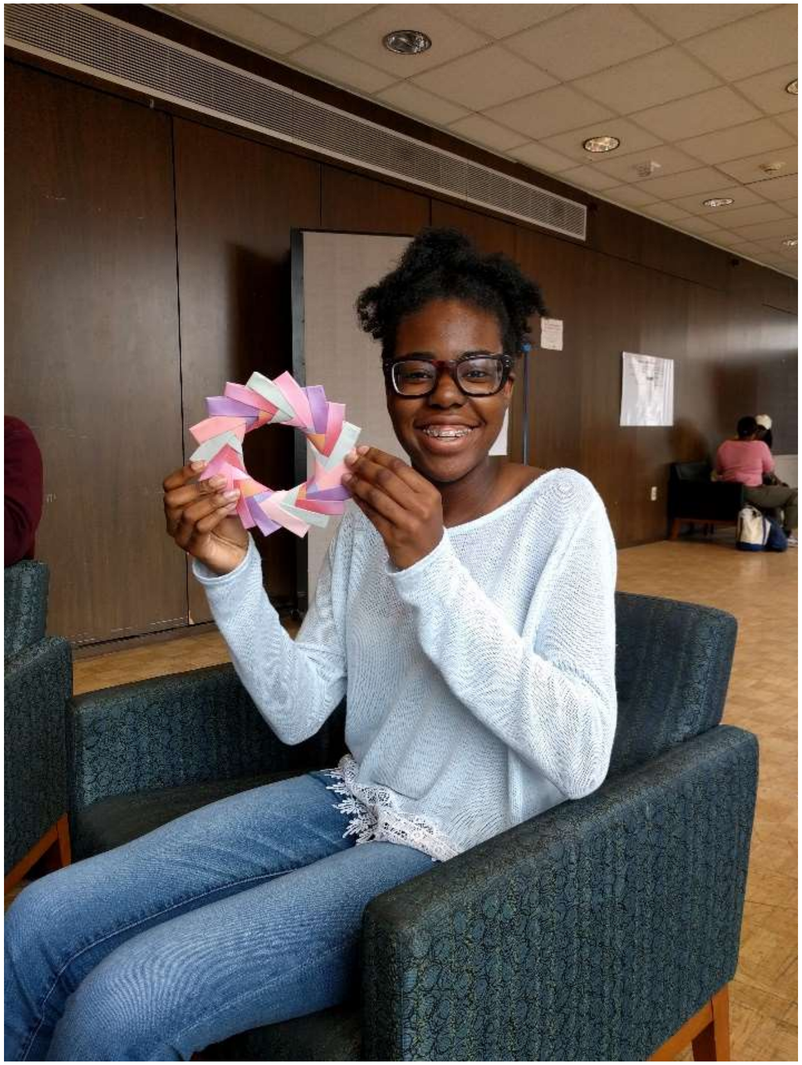
After School Math Options

Foundations: Math Team



Enrichment: Math Circle

(you might ask)
I don’t know all this math.
Can I still teach this way?

Yes.

1. Give problems with existing answer keys (e.g., AMC 8, AMC 10).
2. Play games.
3. Pick problems with many solutions and/or routes.

Learn from your students' ideas!

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Years 1-2:

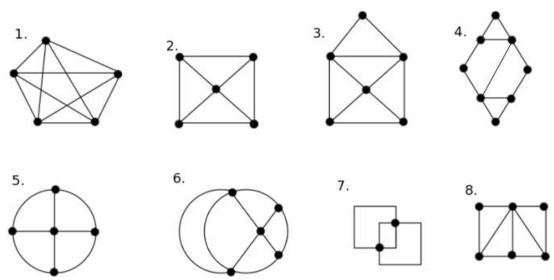
Famous Problems in Mathematics: The Seven Bridges of Konigsberg

Name______________

Period____ Date__________

Part 1: Do Now

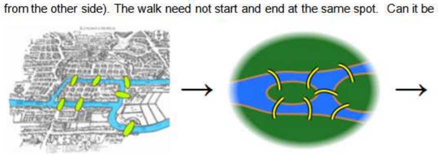
How many of these shapes can you trace *completely*, *without* taking your pencil off the paper or retracing any part twice?



Part 2: The Seven Bridges of Konigsberg

The city of Königsberg in Prussia (now Kaliningrad, Russia) was set on both sides of the Pregel River, and included two large islands which were connected to each other and the mainland by seven bridges.

The problem is to find a walk through the city that crosses each bridge once and only once. The islands cannot be reached by any route other than the bridges, and every bridge must be crossed completely every time. (You can't walk halfway onto the bridge and then turn around and later cross the other half from the other side). The walk need not start and end at the same spot. Can it be done?



Draw a graph of this situation. Can it be solved?

Part 3: Graph Theory Vocabulary

Define the following:

Node:

Edge:

Degree of a node:

Graph:

Path:

Circuit:

Eulerian Path / Eulerian Circuit:

Part 4: A General Solution

Fill out the table below for drawings 1-8 on the front and the Konigsberg

Diagram	Number of nodes, total	Number of edges, total	Number of nodes with ***odd*** degree	Number of nodes with ***even*** degree	Eulerian Path, Eulerian Circuit, or neither?
#1					
#2					
#3					
#4					
#5					
#6					
#7					
#8					
Konigsberg					

How can you tell if a graph will have an Eulerian Path or an Eulerian Circuit? Why does this make sense? (Hint: only ONE of the columns in the above table matters! Which one, and why?)

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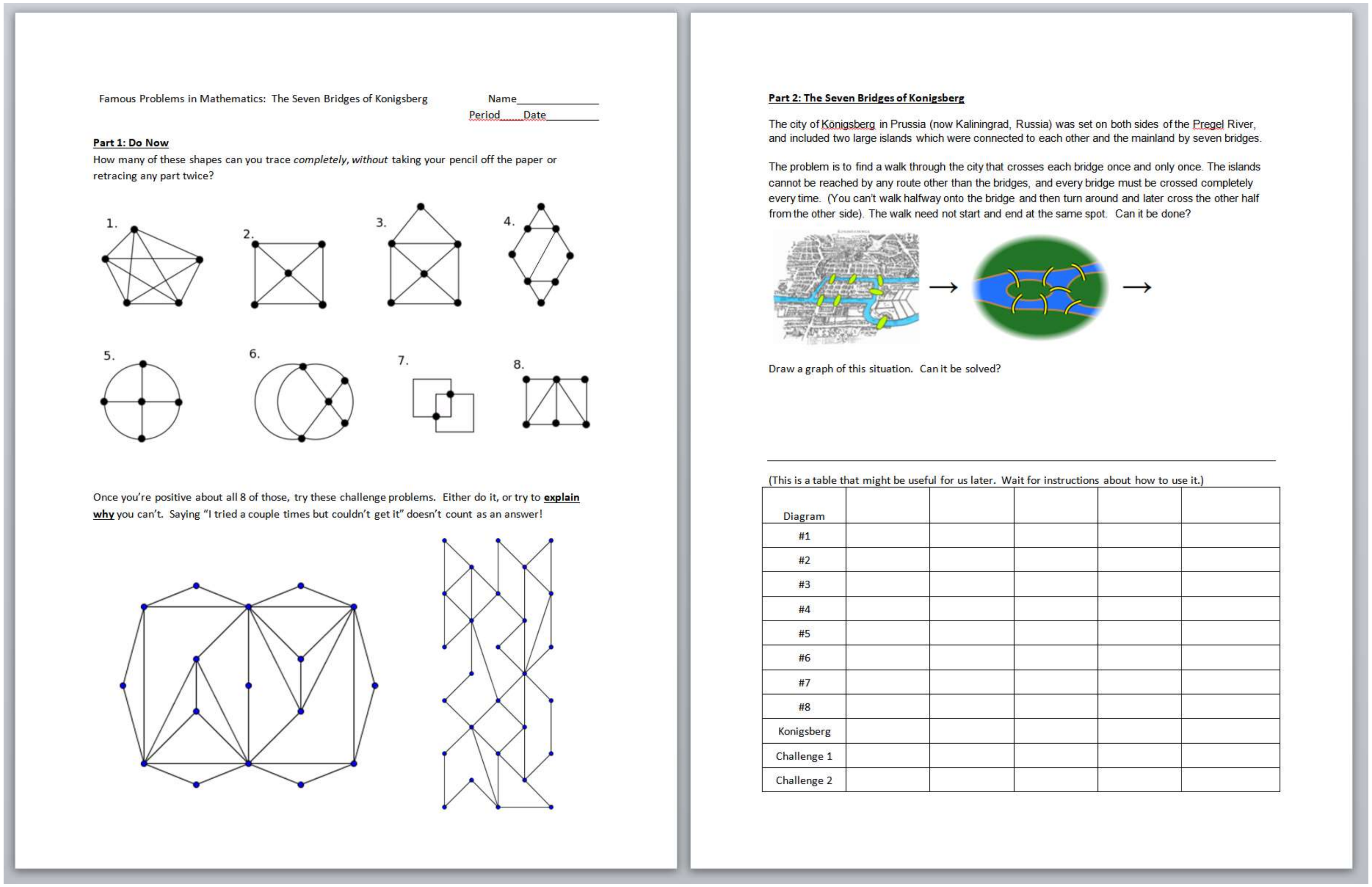
Years 3-5:



Famous Problems in Mathematics: The Seven Bridges of Konigsberg Name____________
Period____ Date__________

Part 1: Do Now

How many of these shapes can you trace *completely*, *without* taking your pencil off the paper or retracing any part twice?

1. 2. 3. 4.

5. 6. 7. 8.

Once you're positive about all 8 of those, try these challenge problems. Either do it, or try to **explain why** you can't. Saying "I tried a couple times but couldn't get it" doesn't count as an answer!

Part 2: The Seven Bridges of Konigsberg

The city of Königsberg in Prussia (now Kaliningrad, Russia) was set on both sides of the Pregel River, and included two large islands which were connected to each other and the mainland by seven bridges.

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Draw a graph of this situation. Can it be solved?

(This is a table that might be useful for us later. Wait for instructions about how to use it.)

Diagram					
#1					
#2					
#3					
#4					
#5					
#6					
#7					
#8					
Konigsberg					
Challenge 1					
Challenge 2					

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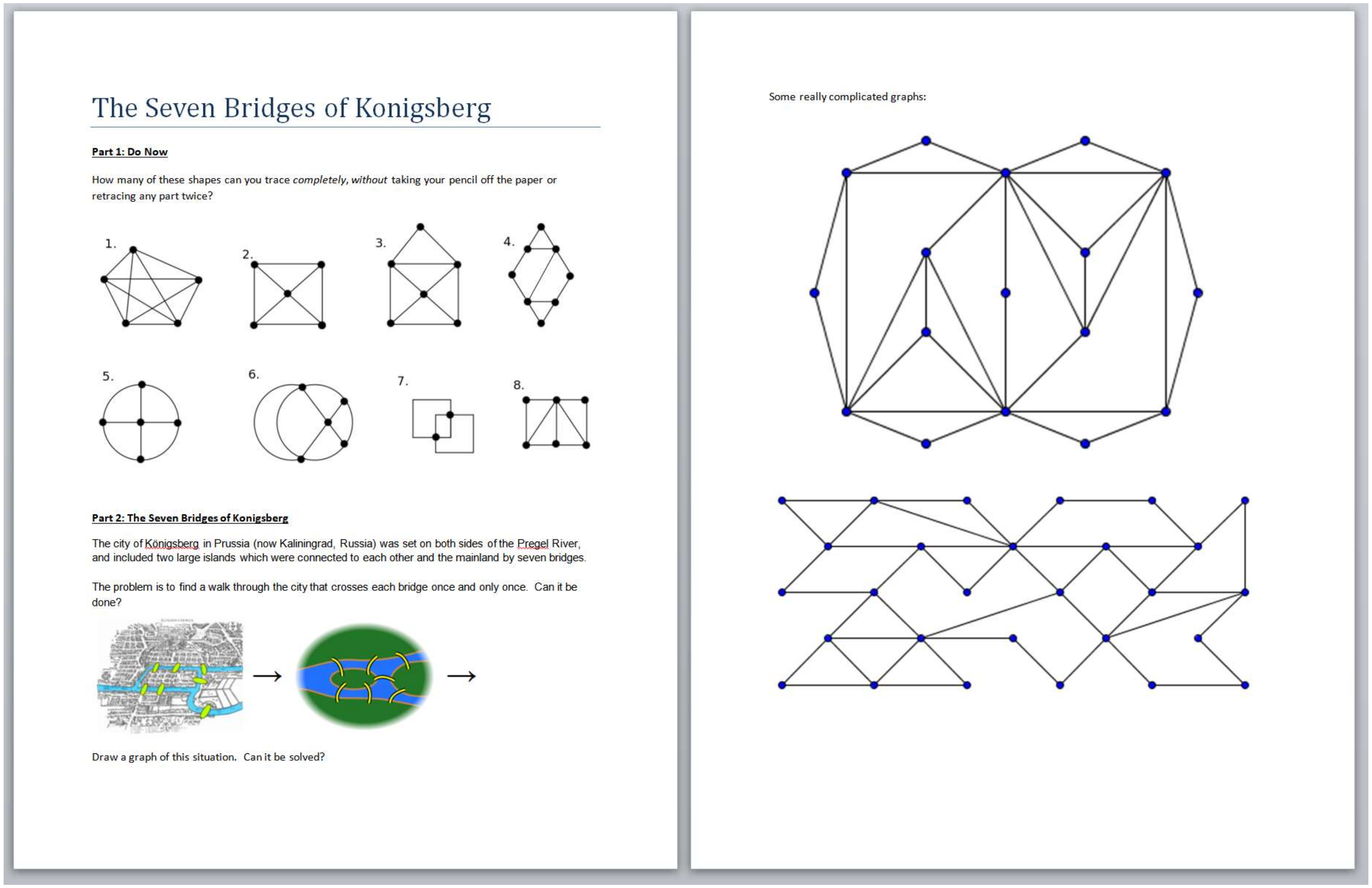
Years 6-7:



The Seven Bridges of Konigsberg

Part 1: Do Now

How many of these shapes can you trace *completely*, *without* taking your pencil off the paper or retracing any part twice?

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Part 2: The Seven Bridges of Konigsberg

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The problem is to find a walk through the city that crosses each bridge once and only once. Can it be done?

→ →

Draw a graph of this situation. Can it be solved?

Some really complicated graphs:

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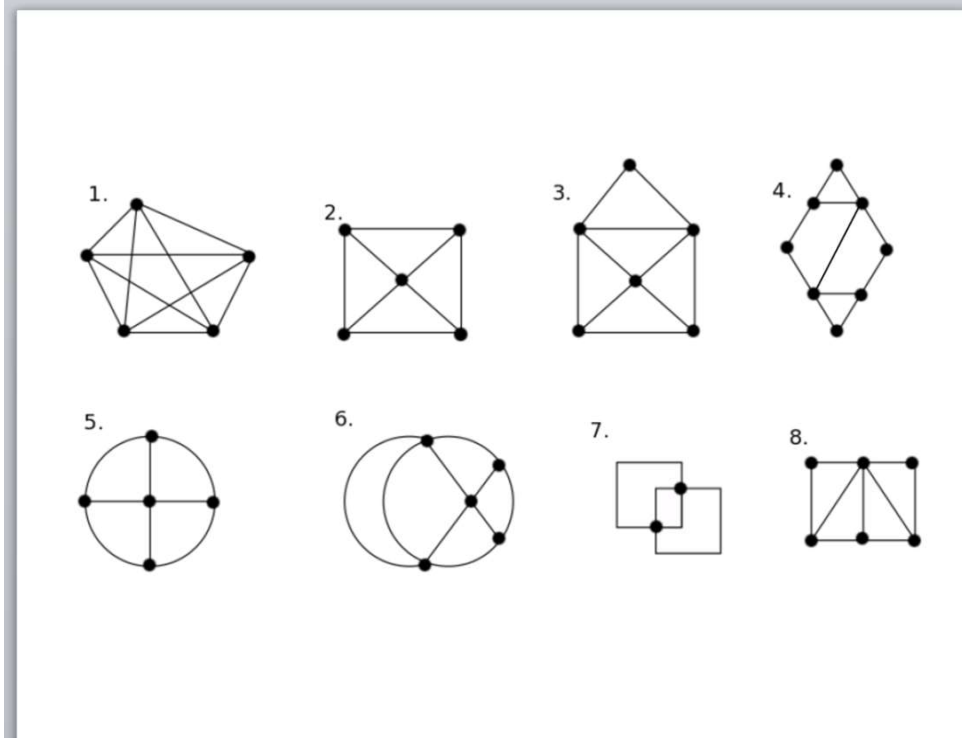
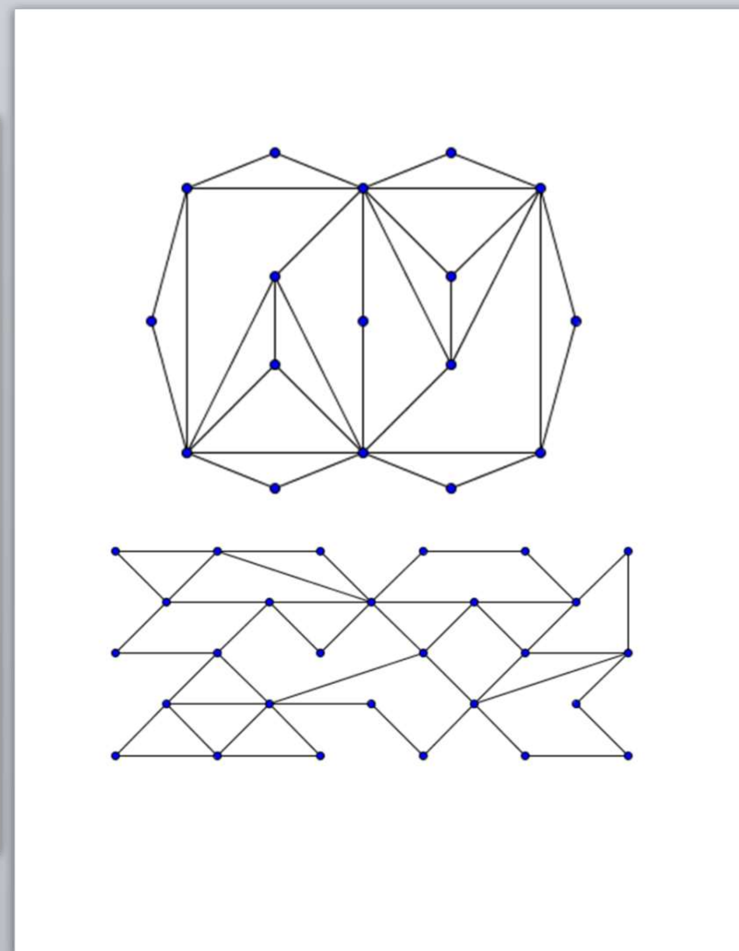
Last weekend:





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(please don’t ask)

When will I use this?

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We learn math for utility but also for joy.

What is math to you?

My favorite Subject. The structure of building and thinking.

Edgar, 8th grade

What is math to you?

Math is the ability to understand the world around you using numbers and ideas.

Camila, 8th grade

What is math to you?

A way I can express my thoughts and talk with and engage in fun arguments with my friends

Alex, 9th grade

What is math to you?

The best subject in the world and most interesting one.

Faith, 12th grade

When you are ready to go down the "rabbit hole"…

Consider what your classroom could look like.

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Down The Rabbit Hole…

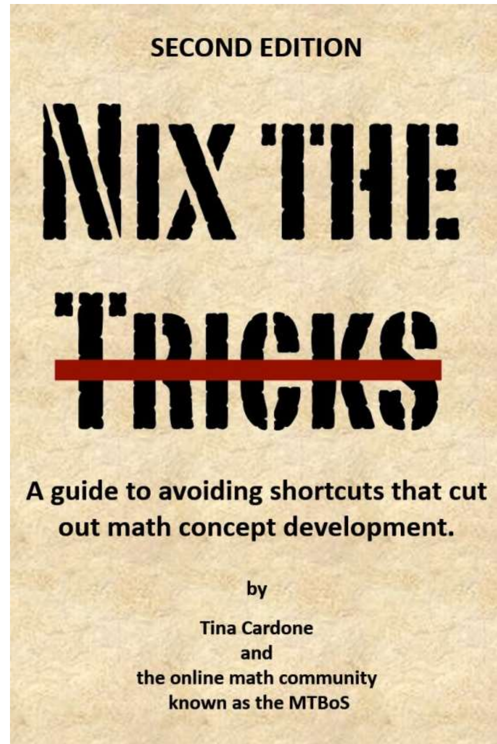
- Math-Twitter-Blog-o-Sphere (MTBoS)

- Dan Meyer: if math is the aspirin, what is the headache?

- Sam Shah: trig identities

Down The Rabbit Hole…

- Nix The Tricks!
- ~~“Cross Multiply”~~
- ~~“FOIL”~~



- Commit to one (two?) per year (semester?)

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REALLY Down The Rabbit Hole...

- Reorder curriculum:

  Explorations* → Abstraction → Rules

- One unit per year, figure out how to get out of the way

*Explorations come in many forms. You might find them in great word problems, rich tasks, or even student questions!

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Questions?

Contact us!

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malcolm.eckel@gmail.com

Download our resources:

beammath.org/NCTM

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